RISK

Risk and the Cost of Capital

▶ **Long before the** development of modern theories linking risk and return, smart financial managers adjusted for risk in capital budgeting. They knew that risky projects are, other things equal, less valuable than safe ones—that is just common sense. Therefore they demanded higher rates of return from risky projects, or they based their decisions about risky projects on conservative forecasts of project cash flows.

Today most companies start with the *company cost of capital* as a benchmark risk-adjusted discount rate for new investments. The company cost of capital is the right discount rate only for investments that have the same risk as the company's overall business. For riskier projects the opportunity cost of capital is greater than the company cost of capital. For safer projects it is less.

The company cost of capital is usually estimated as a weighted-average cost of capital, that is, as the average rate of return demanded by investors in the company's debt and equity. The hardest part of estimating the weighted-average cost of capital is figuring out the cost of equity, that is, the expected rate of return to investors in the firm's common stock. Many firms turn to the capital asset pricing model (CAPM) for an answer. The CAPM states that the expected rate of return equals the risk-free interest rate plus a risk premium that depends on beta and the market risk premium.

We explained the CAPM in the last chapter, but didn't show you how to estimate betas. You can't look up betas in a newspaper or see them clearly by tracking a few day-to-day changes in stock price. But you can get useful statistical estimates from the history of stock and market returns.

Now suppose you're responsible for a specific investment project. How do you know if the project is average risk or above- or below-average risk? We suggest you check whether the project's cash flows are more or less sensitive to the business cycle than the average project. Also check whether the project has higher or lower fixed operating costs (higher or lower operating leverage) and whether it requires large future investments.

Remember that a project's cost of capital depends only on market risk. Diversifiable risk can affect project cash flows but does not increase the cost of capital. Also don't be tempted to add arbitrary fudge factors to discount rates. Fudge factors are too often added to discount rates for projects in unstable parts of the world, for example.

Risk varies from project to project. Risk can also vary over time for a given project. For example, some projects are riskier in youth than in old age. But financial managers usually assume that project risk will be the same in every future period, and they use a single risk-adjusted discount rate for all future cash flows. We close the chapter by introducing certainty equivalents, which illustrate how risk can change over time.

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The **company cost of capital** is defined as the expected return on a portfolio of all the company's existing securities. It is the opportunity cost of capital for investment in the firm's assets, and therefore the appropriate discount rate for the firm's average-risk projects.

If the firm has no debt outstanding, then the company cost of capital is just the expected rate of return on the firm's stock. Many large, successful companies pretty well fit this special case, including Johnson & Johnson (J&J). In Table 8.2 we estimated that investors require a return of 3.8% from J&J common stock. If J&J is contemplating an expansion of its existing business, it would make sense to discount the forecasted cash flows at 3.8%.[1]

The company cost of capital is *not* the correct discount rate if the new projects are more or less risky than the firm's existing business. Each project should in principle be evaluated at its *own* opportunity cost of capital. This is a clear implication of the value-additivity principle introduced in Chapter 7. For a firm composed of assets A and B, the firm value is

$$\text{Firm value} = \text{PV(AB)} = \text{PV(A)} + \text{PV(B)}$$
$$= \text{sum of separate asset values}$$

Here PV(A) and PV(B) are valued just as if they were mini-firms in which stockholders could invest directly. Investors would value A by discounting its forecasted cash flows at a rate reflecting the risk of A. They would value B by discounting at a rate reflecting the risk of B. The two discount rates will, in general, be different. If the present value of an asset depended on the identity of the company that bought it, present values would *not* add up, and we know they do add up. (Consider a portfolio of $1 million invested in J&J and $1 million invested in Toyota. Would any reasonable investor say that the portfolio is worth anything more or less than $2 million?)
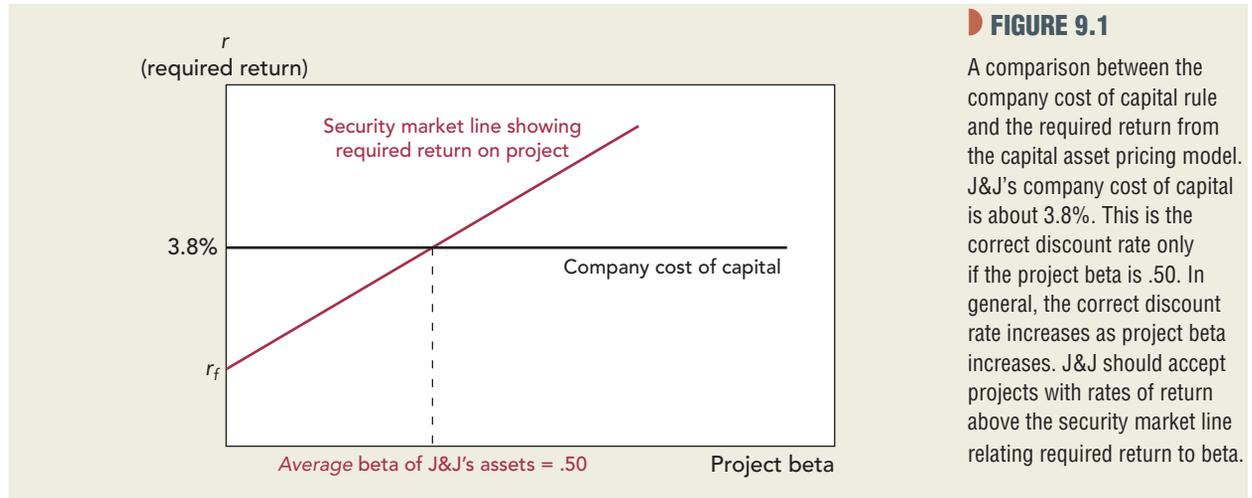
If the firm considers investing in a third project C, it should also value C as if C were a mini-firm. That is, the firm should discount the cash flows of C at the expected rate of return that investors would demand if they could make a separate investment in C. *The opportunity cost of capital depends on the use to which that capital is put.*

Perhaps we're saying the obvious. Think of J&J: it is a massive health care and consumer products company, with $64 billion in sales in 2008. J&J has well-established consumer products, including Band-Aid® bandages, Tylenol®, and products for skin care and babies. It also invests heavily in much chancier ventures, such as biotech research and development (R&D). Do you think that a new production line for baby lotion has the same cost of capital as an investment in biotech R&D? We don't, though we admit that estimating the cost of capital for biotech R&D could be challenging.

Suppose we measure the risk of each project by its beta. Then J&J should accept any project lying above the upward-sloping security market line that links expected return to risk in Figure 9.1. If the project is high-risk, J&J needs a higher prospective return than if the project is low-risk. That is different from the company cost of capital rule, which accepts any project *regardless of its risk* as long as it offers a higher return than the *company's* cost of capital. The rule tells J&J to accept any project above the horizontal cost of capital line in Figure 9.1, that is, any project offering a return of more than 3.8%.

It is clearly silly to suggest that J&J should demand the same rate of return from a very safe project as from a very risky one. If J&J used the company cost of capital rule, it would reject many good low-risk projects and accept many poor high-risk projects. It is also silly to

[1] If 3.8% seems like a very low number, recall that short-term interest rates were at historic lows in 2009. Long-term interest rates were higher, and J&J probably would use a higher discount rate for cash flows spread out over many future years. We return to this distinction later in the chapter. We have also simplified by treating J&J as all-equity-financed. J&J's market-value debt ratio is very low, but not zero. We discuss debt financing and the weighted-average cost of capital below.

A comparison between the company cost of capital rule and the required return from the capital asset pricing model. J&J's company cost of capital is about 3.8%. This is the correct discount rate only if the project beta is .50. In general, the correct discount rate increases as project beta increases. J&J should accept projects with rates of return above the security market line relating required return to beta.

suggest that just because another company has a low company cost of capital, it is justified in accepting projects that J&J would reject.

Perfect Pitch and the Cost of Capital

The true cost of capital depends on project risk, not on the company undertaking the project. So why is so much time spent estimating the company cost of capital?

There are two reasons. First, many (maybe most) projects can be treated as average risk, that is, neither more nor less risky than the average of the company's other assets. For these projects the company cost of capital is the right discount rate. Second, the company cost of capital is a useful starting point for setting discount rates for unusually risky or safe projects. It is easier to add to, or subtract from, the company cost of capital than to estimate each project's cost of capital from scratch.

There is a good musical analogy here. Most of us, lacking perfect pitch, need a well-defined reference point, like middle C, before we can sing on key. But anyone who can carry a tune gets *relative* pitches right. Businesspeople have good intuition about *relative* risks, at least in industries they are used to, but not about absolute risk or required rates of return. Therefore, they set a companywide cost of capital as a benchmark. This is not the right discount rate for everything the company does, but adjustments can be made for more or less risky ventures.

That said, we have to admit that many large companies use the company cost of capital not just as a benchmark, but also as an all-purpose discount rate for every project proposal. Measuring differences in risk is difficult to do objectively, and financial managers shy away from intracorporate squabbles. (You can imagine the bickering: "My projects are safer than yours! I want a lower discount rate!" "No they're not! Your projects are riskier than a naked call option!")[2]

When firms force the use of a single company cost of capital, risk adjustment shifts from the discount rate to project cash flows. Top management may demand extra-conservative cash-flow forecasts from extra-risky projects. They may refuse to sign off on an extra-risky project unless NPV, computed at the company cost of capital, is well above zero. Rough-and-ready risk adjustments are better than none at all.

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[2] A "naked" call option is an option purchased with no offsetting (hedging) position in the underlying stock or in other options. We discuss options in Chapter 20.

Debt and the Company Cost of Capital

We defined the company cost of capital as "the expected return on a portfolio of all the company's existing securities." That portfolio usually includes debt as well as equity. Thus the cost of capital is estimated as a blend of the *cost of debt* (the interest rate) and the *cost of equity* (the expected rate of return demanded by investors in the firm's common stock).

Suppose the company's market-value balance sheet looks like this:

Asset value	100	Debt	$D = 30$ at 7.5%
		Equity	$E = 70$ at 15%
Asset value	100	Firm value	$V = 100$

The values of debt and equity add up to overall firm value ($D + E = V$) and firm value V equals asset value. These figures are all market values, not book (accounting) values. The market value of equity is often much larger than the book value, so the market debt ratio D/V is often much lower than a debt ratio computed from the book balance sheet.

The 7.5% cost of debt is the opportunity cost of capital for the investors who hold the firm's debt. The 15% cost of equity is the opportunity cost of capital for the investors who hold the firm's shares. Neither measures the *company* cost of capital, that is, the opportunity cost of investing in the firm's *assets*. The cost of debt is less than the company cost of capital, because debt is safer than the assets. The cost of equity is greater than the company cost of capital, because the equity of a firm that borrows is riskier than the assets. Equity is not a direct claim on the firm's free cash flow. It is a residual claim that stands behind debt.

The company cost of capital is not equal to the cost of debt or to the cost of equity but is a blend of the two. Suppose you purchased a portfolio consisting of 100% of the firm's debt and 100% of its equity. Then you would own 100% of its assets lock, stock, and barrel. You would not share the firm's free cash flow with anyone; every dollar that the firm pays out would be paid to you.

The expected rate of return on your hypothetical portfolio is the company cost of capital. The expected rate of return is just a weighted average of the cost of debt ($r_D = 7.5\%$) and the cost of equity ($r_E = 15\%$). The weights are the relative market values of the firm's debt and equity, that is, $D/V = 30\%$ and $E/V = 70\%$.[3]

$$\text{Company cost of capital} = r_D D/V + r_E E/V$$
$$= 7.5 \times .30 + 15 \times .70 = 12.75\%$$

This blended measure of the company cost of capital is called the **weighted-average cost of capital** or **WACC** (pronounced "whack"). Calculating WACC is a bit more complicated than our example suggests, however. For example, interest is a tax-deductible expense for corporations, so the after-tax cost of debt is $(1 - T_c)r_D$, where T_c is the marginal corporate tax rate. Suppose $T_c = 35\%$. Then *after-tax WACC* is

$$\text{After-tax WACC} = (1 - T_c)r_D D/V + r_E E/V$$
$$= (1 - .35) \times 7.5 \times .30 + 15 \times .70 = 12.0\%$$

We give another example of the after-tax WACC later in this chapter, and we cover the topic in much more detail in Chapter 19. But now we turn to the hardest part of calculating WACC, estimating the cost of equity.

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[3] Recall that the 30% and 70% weights in your hypothetical portfolio are based on market, not book, values. Now you can see why. If the portfolio were constructed with different book weights, say 50-50, then the portfolio returns could not equal the asset returns.

9-2   Measuring the Cost of Equity

To calculate the weighted-average cost of capital, you need an estimate of the cost of equity. You decide to use the capital asset pricing model (CAPM). Here you are in good company: as we saw in the last chapter, most large U.S. companies do use the CAPM to estimate the cost of equity, which is the expected rate of return on the firm's common stock.[4] The CAPM says that

$$\text{Expected stock return} = r_f + \beta\,(r_m - r_f)$$

Now you have to estimate beta. Let us see how that is done in practice.

Estimating Beta

In principle we are interested in the future beta of the company's stock, but lacking a crystal ball, we turn first to historical evidence. For example, look at the scatter diagram at the top left of Figure 9.2. Each dot represents the return on Amazon stock and the return on the market in a particular month. The plot starts in January 1999 and runs to December 2003, so there are 60 dots in all.

The second diagram on the left shows a similar plot for the returns on Disney stock, and the third shows a plot for Campbell Soup. In each case we have fitted a line through the points. The slope of this line is an estimate of beta.[5] It tells us how much on average the stock price changed when the market return was 1% higher or lower.

The right-hand diagrams show similar plots for the same three stocks during the subsequent period ending in December 2008. Although the slopes varied from the first period to the second, there is little doubt that Campbell Soup's beta is much less than Amazon's or that Disney's beta falls somewhere between the two. If you had used the past beta of each stock to predict its future beta, you would not have been too far off.

Only a small portion of each stock's total risk comes from movements in the market. The rest is firm-specific, diversifiable risk, which shows up in the scatter of points around the fitted lines in Figure 9.2. *R-squared* (R^2) measures the proportion of the total variance in the stock's returns that can be explained by market movements. For example, from 2004 to 2008, the R^2 for Disney was .395. In other words, about 40% of Disney's risk was market risk and 60% was diversifiable risk. The variance of the returns on Disney stock was 383.[6] So we could say that the variance in stock returns that was due to the market was $.4 \times 383 = 153$, and the variance of diversifiable returns was $.6 \times 383 = 230$.

The estimates of beta shown in Figure 9.2 are just that. They are based on the stocks' returns in 60 particular months. The noise in the returns can obscure the true beta.[7] Therefore, statisticians calculate the *standard error* of the estimated beta to show the extent of possible mismeasurement. Then they set up a *confidence interval* of the estimated value plus or minus two standard errors. For example, the standard error of Disney's

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[4] The CAPM is not the last word on risk and return, of course, but the principles and procedures covered in this chapter work just as well with other models such as the Fama–French three-factor model. See Section 8-4.

[5] Notice that to estimate beta you must regress the *returns* on the stock on the market *returns*. You would get a very similar estimate if you simply used the percentage *changes* in the stock price and the market index. But sometimes people make the mistake of regressing the stock price *level* on the *level* of the index and obtain nonsense results.

[6] This is an annual figure; we annualized the monthly variance by multiplying by 12 (see footnote 18 in Chapter 7). The standard deviation was $\sqrt{383} = 19.6\%$.

[7] Estimates of beta may be distorted if there are extreme returns in one or two months. This is a potential problem in our estimates for 2004–2008, since you can see in Figure 9.2 that there was one month (October 2008) when the market fell by over 16%. The performance of each stock that month has an excessive effect on the estimated beta. In such cases statisticians may prefer to give less weight to the extreme observations or even to omit them entirely.
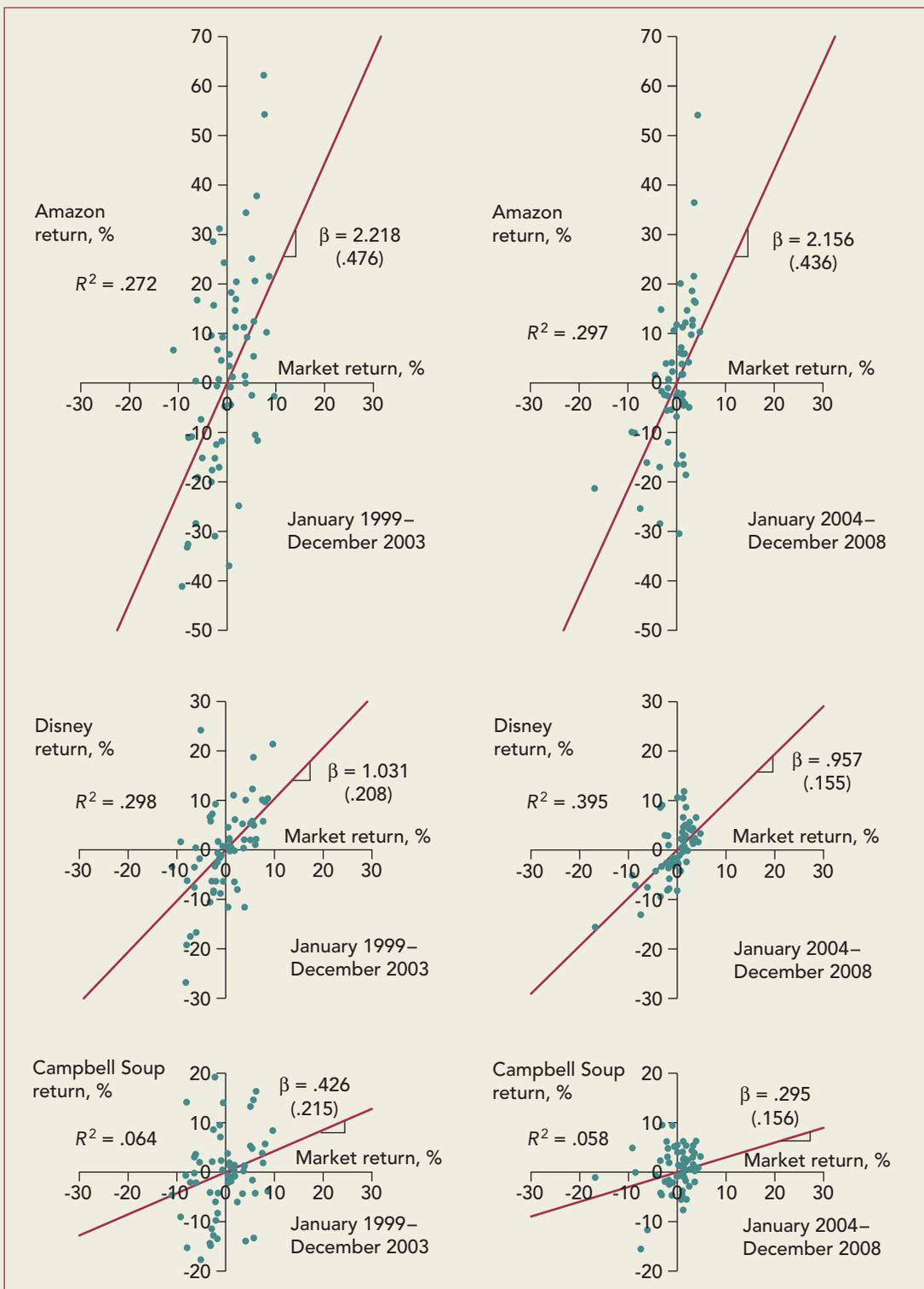
FIGURE 9.2

We have used past returns to estimate the betas of three stocks for the periods January 1999 to December 2003 (left-hand diagrams) and January 2004 to December 2008 (right-hand diagrams). Beta is the slope of the fitted line. Notice that in both periods Amazon had the highest beta and Campbell Soup the lowest. Standard errors are in parentheses below the betas. The standard error shows the range of possible error in the beta estimate. We also report the proportion of total risk that is due to market movements (R^2).

estimated beta in the most recent period is about .16. Thus the confidence interval for Disney's beta is .96 plus or minus 2 × .16. If you state that the *true* beta for Disney is between .64 and 1.28, you have a 95% chance of being right. Notice that we can be equally confident of our estimate of Campbell Soup's beta, but much less confident of Amazon's.

Usually you will have more information (and thus more confidence) than this simple, and somewhat depressing, calculation suggests. For example, you know that Campbell Soup's estimated beta was well below 1 in two successive five-year periods. Amazon's estimated beta was well above 1 in both periods. Nevertheless, there is always a large margin for error when estimating the beta for individual stocks.

Fortunately, the estimation errors tend to cancel out when you estimate betas of *portfolios*.[8] That is why financial managers often turn to *industry betas*. For example, Table 9.1 shows estimates of beta and the standard errors of these estimates for the common stocks of six large railroad companies. Five of the standard errors are above .2. Kansas City Southern's is .29, large enough to preclude a price estimate of that railroad's beta. However, the table also shows the estimated beta for a portfolio of all six railroad stocks. Notice that the estimated industry beta is somewhat more reliable. This shows up in the lower standard error.

The Expected Return on Union Pacific Corporation's Common Stock

Suppose that in early 2009 you had been asked to estimate the company cost of capital of Union Pacific. Table 9.1 provides two clues about the true beta of Union Pacific's stock: the direct estimate of 1.16 and the average estimate for the industry of 1.24. We will use the direct estimate of 1.16.[9]

The next issue is what value to use for the risk-free interest rate. By the first months of 2009, the U.S. Federal Reserve Board had pushed down Treasury bill rates to about .2% in an attempt to reverse the financial crisis and recession. The one-year interest rate was only a little higher, at about .7%. Yields on longer-maturity U.S. Treasury bonds were higher still, at about 3.3% on 20-year bonds.

The CAPM is a short-term model. It works period by period and calls for a short-term interest rate. But could a .2% three-month risk-free rate give the right discount rate for cash flows 10 or 20 years in the future? Well, now that you mention it, probably not.

Financial managers muddle through this problem in one of two ways. The first way simply uses a long-term risk-free rate in the CAPM formula. If this short-cut is used, then

	Beta	Standard Error
Burlington Northern Santa Fe	1.01	.19
Canadian Pacific	1.34	.23
CSX	1.14	.22
Kansas City Southern	1.75	.29
Norfolk Southern	1.05	.24
Union Pacific	1.16	.21
Industry portfolio	1.24	.18

▶ **TABLE 9.1**   Estimates of betas and standard errors for a sample of large railroad companies and for an equally weighted portfolio of these companies, based on monthly returns from January 2004 to December 2008. The portfolio beta is more reliable than the betas of the individual companies. Note the lower standard error for the portfolio.

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[8] If the observations are independent, the standard error of the estimated mean beta declines in proportion to the square root of the number of stocks in the portfolio.

[9] One reason that Union Pacific's beta is less than that of the average railroad is that the company has below-average debt ratio. Chapter 19 explains how to adjust betas for differences in debt ratios.

the market risk premium must be restated as the average difference between market returns and returns on *long-term* Treasuries.[10]

The second way retains the usual definition of the market risk premium as the difference between market returns and returns on *short-term* Treasury bill rates. But now you have to forecast the expected return from holding Treasury bills over the life of the project. In Chapter 3 we observed that investors require a risk premium for holding long-term bonds rather than bills. Table 7.1 showed that over the past century this risk premium has averaged about 1.5%. So to get a rough but reasonable estimate of the expected long-term return from investing in Treasury bills, we need to subtract 1.5% from the current yield on long-term bonds. In our example

$$\text{Expected long-term return from bills} = \text{yield on long-term bonds} - 1.5\%$$
$$= 3.3 - 1.5 = 1.8\%$$

This is a plausible estimate of the expected average future return on Treasury bills. We therefore use this rate in our example.

Returning to our Union Pacific example, suppose you decide to use a market risk premium of 7%. Then the resulting estimate for Union Pacific's cost of equity is about 9.9%:

$$\text{Cost of equity} = \text{expected return} = r_f + \beta(r_m - r_f)$$
$$= 1.8 + 1.16 \times 7.0 = 9.9\%$$

Union Pacific's After-Tax Weighted-Average Cost of Capital

Now you can calculate Union Pacific's after-tax WACC in early 2009. The company's cost of debt was about 7.8%. With a 35% corporate tax rate, the after-tax cost of debt was $r_D(1 - Tc) = 7.8 \times (1 - .35) = 5.1\%$. The ratio of debt to overall company value was $D/V = 31.5\%$. Therefore:

$$\text{After-tax WACC} = (1 - T_c)r_D D/V + r_E E/V$$
$$= (1 - .35) \times 7.8 \times .315 + 9.9 \times .685 = 8.4\%$$

Union Pacific should set its overall cost of capital to 8.4%, assuming that its CFO agrees with our estimates.

Warning   The cost of debt is always less than the cost of equity. The WACC formula blends the two costs. The formula is dangerous, however, because it suggests that the average cost of capital could be reduced by substituting cheap debt for expensive equity. It doesn't work that way! As the debt ratio D/V increases, the cost of the remaining equity also increases, offsetting the apparent advantage of more cheap debt. We show how and why this offset happens in Chapter 17.

Debt does have a tax advantage, however, because interest is a tax-deductible expense. That is why we use the after-tax cost of debt in the after-tax WACC. We cover debt and taxes in much more detail in Chapters 18 and 19.

Union Pacific's Asset Beta

The after-tax WACC depends on the average risk of the company's assets, but it also depends on taxes and financing. It's easier to think about project risk if you measure it directly. The direct measure is called the **asset beta.**

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[10] This approach gives a security market line with a higher intercept and a lower market risk premium. Using a "flatter" security market line is perhaps a better match to the historical evidence, which shows that the slope of average returns against beta is not as steeply upward-sloping as the CAPM predicts. See Figures 8.8 and 8.9.

We calculate the asset beta as a blend of the separate betas of debt (β_D) and equity (β_E). For Union Pacific we have $\beta_E = 1.16$, and we'll assume $\beta_D = .3$.[11] The weights are the fractions of debt and equity financing, $D/V = .315$ and $E/V = .685$:

$$\text{Asset beta} = \beta_A = \beta_D(D/V) + \beta_E(E/V)$$
$$\beta_A = .3 \times .315 + 1.16 \times .685 = .89$$

Calculating an asset beta is similar to calculating a weighted-average cost of capital. The debt and equity weights D/V and E/V are the same. The logic is also the same: Suppose you purchased a portfolio consisting of 100% of the firm's debt and 100% of its equity. Then you would own 100% of its assets lock, stock, and barrel, and the beta of your portfolio would equal the beta of the assets. The portfolio beta is of course just a weighted average of the betas of debt and equity.

This asset beta is an estimate of the average risk of Union Pacific's railroad business. It is a useful benchmark, but it can take you only so far. Not all railroad investments are average risk. And if you are the first to use railroad-track networks as interplanetary transmission antennas, you will have no asset beta to start with.

How can you make informed judgments about costs of capital for projects or lines of business when you suspect that risk is *not* average? That is our next topic.

9-3 Analyzing Project Risk

Suppose that a coal-mining corporation wants to assess the risk of investing in commercial real estate, for example, in a new company headquarters. The asset beta for coal mining is not helpful. You need to know the beta of real estate. Fortunately, portfolios of commercial real estate are traded. For example, you could estimate asset betas from returns on Real Estate Investment Trusts (REITs) specializing in commercial real estate.[12] The REITs would serve as traded *comparables* for the proposed office building. You could also turn to indexes of real estate prices and returns derived from sales and appraisals of commercial properties.[13]

A company that wants to set a cost of capital for one particular line of business typically looks for *pure plays* in that line of business. Pure-play companies are public firms that specialize in one activity. For example, suppose that J&J wants to set a cost of capital for its pharmaceutical business. It could estimate the average asset beta or cost of capital for pharmaceutical companies that have *not* diversified into consumer products like Band-Aid® bandages or baby powder.

Overall company costs of capital are almost useless for *conglomerates*. Conglomerates diversify into several unrelated industries, so they have to consider industry-specific costs of capital. They therefore look for pure plays in the relevant industries. Take Richard Branson's Virgin Group as an example. The group combines many different companies, including airlines (Virgin Atlantic) and retail outlets for music, books, and movies (Virgin Megastores). Fortunately there are many examples of pure-play airlines and pure-play retail

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[11]Why is the debt beta positive? Two reasons: First, debt investors worry about the risk of default. Corporate bond prices fall, relative to Treasury-bond prices, when the economy goes from expansion to recession. The risk of default is therefore partly a macroeconomic and market risk. Second, all bonds are exposed to uncertainty about interest rates and inflation. Even Treasury bonds have positive betas when long-term interest rates and inflation are volatile and uncertain.

[12] REITs are investment funds that invest in real estate. You would have to be careful to identify REITs investing in commercial properties similar to the proposed office building. There are also REITs that invest in other types of real estate, including apartment buildings, shopping centers, and timberland.

[13] See Chapter 23 in D. Geltner, N. G. Miller, J. Clayton, and P. Eichholtz, *Commercial Real Estate Analysis and Investments,* 2nd ed. (South-Western College Publishing, 2006).

chains. The trick is picking the comparables with business risks that are most similar to Virgin's companies.

Sometimes good comparables are not available or not a good match to a particular project. Then the financial manager has to exercise his or her judgment. Here we offer the following advice:

1. *Think about the determinants of asset betas.* Often the characteristics of high- and low-beta assets can be observed when the beta itself cannot be.
2. *Don't be fooled by diversifiable risk.*
3. *Avoid fudge factors.* Don't give in to the temptation to add fudge factors to the discount rate to offset things that could go wrong with the proposed investment. Adjust cash-flow forecasts first.

What Determines Asset Betas?

Cyclicality   Many people's intuition associates risk with the variability of earnings or cash flow. But much of this variability reflects diversifiable risk. Lone prospectors searching for gold look forward to extremely uncertain future income, but whether they strike it rich is unlikely to depend on the performance of the market portfolio. Even if they do find gold, they do not bear much market risk. Therefore, an investment in gold prospecting has a high standard deviation but a relatively low beta.

What really counts is the strength of the relationship between the firm's earnings and the aggregate earnings on all real assets. We can measure this either by the *earnings beta* or by the *cash-flow beta*. These are just like a real beta except that changes in earnings or cash flow are used in place of rates of return on securities. We would predict that firms with high earnings or cash-flow betas should also have high asset betas.

This means that cyclical firms—firms whose revenues and earnings are strongly dependent on the state of the business cycle—tend to be high-beta firms. Thus you should demand a higher rate of return from investments whose performance is strongly tied to the performance of the economy. Examples of cyclical businesses include airlines, luxury resorts and restaurants, construction, and steel. (Much of the demand for steel depends on construction and capital investment.) Examples of less-cyclical businesses include food and tobacco products and established consumer brands such as J&J's baby products. MBA programs are another example, because spending a year or two at a business school is an easier choice when jobs are scarce. Applications to top MBA programs increase in recessions.

Operating Leverage   A production facility with high fixed costs, relative to variable costs, is said to have high *operating leverage.* High operating leverage means a high asset beta. Let us see how this works.

The cash flows generated by an asset can be broken down into revenue, fixed costs, and variable costs:

$$\text{Cash flow} = \text{revenue} - \text{fixed cost} - \text{variable cost}$$

Costs are variable if they depend on the rate of output. Examples are raw materials, sales commissions, and some labor and maintenance costs. Fixed costs are cash outflows that occur regardless of whether the asset is active or idle, for example, property taxes or the wages of workers under contract.

We can break down the asset's present value in the same way:

$$\text{PV(asset)} = \text{PV(revenue)} - \text{PV(fixed cost)} - \text{PV(variable cost)}$$

Or equivalently

$$PV(revenue) = PV(fixed\ cost) + PV(variable\ cost) + PV(asset)$$

Those who *receive* the fixed costs are like debtholders in the project; they simply get a fixed payment. Those who receive the net cash flows from the asset are like holders of common stock; they get whatever is left after payment of the fixed costs.

We can now figure out how the asset's beta is related to the betas of the values of revenue and costs. The beta of PV(revenue) is a weighted average of the betas of its component parts:

$$\beta_{revenue} = \beta_{fixed\ cost} \frac{PV(fixed\ cost)}{PV(revenue)}$$

$$+ \beta_{variable\ cost} \frac{PV(variable\ cost)}{PV(revenue)} + \beta_{assets} \frac{PV(asset)}{PV(revenue)}$$

The fixed-cost beta should be about zero; whoever receives the fixed costs receives a fixed stream of cash flows. The betas of the revenues and variable costs should be approximately the same, because they respond to the same underlying variable, the rate of output. Therefore we can substitute $\beta_{revenue}$ for $\beta_{variable\ cost}$ and solve for the asset beta. Remember, we are assuming $\beta_{fixed\ cost} = 0$. Also, $PV(revenue) - PV(variable\ cost) = PV(asset) + PV(fixed\ cost)$.[14]

$$\beta_{assets} = \beta_{revenue} \frac{PV(revenue) - PV(variable\ cost)}{PV(asset)}$$

$$= \beta_{revenue} \left[1 + \frac{PV(fixed\ cost)}{PV(asset)} \right]$$

Thus, given the cyclicality of revenues (reflected in $\beta_{revenue}$), the asset beta is proportional to the ratio of the present value of fixed costs to the present value of the project.

Now you have a rule of thumb for judging the relative risks of alternative designs or technologies for producing the same project. Other things being equal, the alternative with the higher ratio of fixed costs to project value will have the higher project beta. Empirical tests confirm that companies with high operating leverage actually do have high betas.[15]

We have interpreted fixed costs as costs of production, but fixed costs can show up in other forms, for example, as future investment outlays. Suppose that an electric utility commits to build a large electricity-generating plant. The plant will take several years to build, and the cost is fixed. Our operating leverage formula still applies, but with PV(future investment) included in PV(fixed costs). The commitment to invest therefore increases the plant's asset beta. Of course PV(future investment) decreases as the plant is constructed and disappears when the plant is up and running. Therefore the plant's asset beta is only temporarily high during construction.

Other Sources of Risk   So far we have focused on cash flows. Cash-flow risk is not the only risk. A project's value is equal to the expected cash flows discounted at the risk-adjusted discount rate *r*. If either the risk-free rate or the market risk premium changes, then *r* will change and so will the project value. A project with very long-term cash flows is more exposed to such

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[14] In Chapter 10 we describe an accounting measure of the degree of operating leverage (DOL), defined as DOL = 1 + fixed costs/profits. DOL measures the percentage change in profits for a 1% change in revenue. We have derived here a version of DOL expressed in PVs and betas.

[15] See B. Lev, "On the Association between Operating Leverage and Risk," *Journal of Financial and Quantitative Analysis* 9 (September 1974), pp. 627–642; and G. N. Mandelker and S. G. Rhee, "The Impact of the Degrees of Operating and Financial Leverage on Systematic Risk of Common Stock," *Journal of Financial and Quantitative Analysis* 19 (March 1984), pp. 45–57.

shifts in the discount rate than one with short-term cash flows. This project will, therefore, have a high beta even though it may not have high operating leverage and cyclicality.[16]

You cannot hope to estimate the relative risk of assets with any precision, but good managers examine any project from a variety of angles and look for clues as to its riskiness. They know that high market risk is a characteristic of cyclical ventures, of projects with high fixed costs and of projects that are sensitive to marketwide changes in the discount rate. They think about the major uncertainties affecting the economy and consider how projects are affected by these uncertainties.

Don't Be Fooled by Diversifiable Risk

In this chapter we have defined risk as the asset beta for a firm, industry, or project. But in everyday usage, "risk" simply means "bad outcome." People think of the risks of a project as a list of things that can go wrong. For example,

- A geologist looking for oil worries about the risk of a dry hole.
- A pharmaceutical-company scientist worries about the risk that a new drug will have unacceptable side effects.
- A plant manager worries that new technology for a production line will fail to work, requiring expensive changes and repairs.
- A telecom CFO worries about the risk that a communications satellite will be damaged by space debris. (This was the fate of an Iridium satellite in 2009, when it collided with Russia's defunct Cosmos 2251. Both were blown to smithereens.)

Notice that these risks are all diversifiable. For example, the Iridium-Cosmos collision was definitely a zero-beta event. These hazards do not affect asset betas and should not affect the discount rate for the projects.

Sometimes financial managers increase discount rates in an attempt to offset these risks. This makes no sense. Diversifiable risks should not increase the cost of capital.

EXAMPLE 9.1   ● Allowing for Possible Bad Outcomes

Project Z will produce just one cash flow, forecasted at $1 million at year 1. It is regarded as average risk, suitable for discounting at a 10% company cost of capital:

$$PV = \frac{C_1}{1 + r} = \frac{1,000,000}{1.1} = \$909,100$$

But now you discover that the company's engineers are behind schedule in developing the technology required for the project. They are confident it will work, but they admit to a small chance that it will not. You still see the *most likely* outcome as $1 million, but you also see some chance that project Z will generate *zero* cash flow next year.

Now the project's prospects are clouded by your new worry about technology. It must be worth less than the $909,100 you calculated before that worry arose. But how much less? There is *some* discount rate (10% plus a fudge factor) that will give the right value, but we do not know what that adjusted discount rate is.

We suggest you reconsider your original $1 million forecast for project Z's cash flow. Project cash flows are supposed to be *unbiased* forecasts that give due weight to all possible outcomes, favorable and unfavorable. Managers making unbiased forecasts are correct on

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[16] See J. Y. Campbell and J. Mei, "Where Do Betas Come From? Asset Price Dynamics and the Sources of Systematic Risk," *Review of Financial Studies* 6 (Fall 1993), pp. 567–592. Cornell discusses the effect of duration on project risk in B. Cornell, "Risk, Duration and Capital Budgeting: New Evidence on Some Old Questions," *Journal of Business* 72 (April 1999), pp. 183–200.

average. Sometimes their forecasts will turn out high, other times low, but their errors will average out over many projects.

If you forecast a cash flow of $1 million for projects like Z, you will overestimate the average cash flow, because every now and then you will hit a zero. Those zeros should be "averaged in" to your forecasts.

For many projects, the most likely cash flow is also the unbiased forecast. If there are three possible outcomes with the probabilities shown below, the unbiased forecast is $1 million. (The unbiased forecast is the sum of the probability-weighted cash flows.)

Possible Cash Flow	Probability	Probability-Weighted Cash Flow	Unbiased Forecast
1.2	.25	.3	
1.0	.50	.5	1.0, or $1 million
.8	.25	.2	

This might describe the initial prospects of project Z. But if technological uncertainty introduces a 10% chance of a zero cash flow, the unbiased forecast could drop to $900,000:

Possible Cash Flow	Probability	Probability-Weighted Cash Flow	Unbiased Forecast
1.2	.225	.27	
1.0	.45	.45	.90, or $900,000
.8	.225	.18	
0	.10	.0	

The present value is

$$PV = \frac{.90}{1.1} = .818, \text{ or } \$818,000$$

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Managers often work out a range of possible outcomes for major projects, sometimes with explicit probabilities attached. We give more elaborate examples and further discussion in Chapter 10. But even when outcomes and probabilities are not explicitly written down, the manager can still consider the good and bad outcomes as well as the most likely one. When the bad outcomes outweigh the good, the cash-flow forecast should be reduced until balance is regained.

Step 1, then, is to do your best to make unbiased forecasts of a project's cash flows. Unbiased forecasts incorporate all risks, including diversifiable risks as well as market risks. Step 2 is to consider whether *diversified* investors would regard the project as more or less risky than the average project. In this step only market risks are relevant.

Avoid Fudge Factors in Discount Rates

Think back to our example of project Z, where we reduced forecasted cash flows from $1 million to $900,000 to account for a possible failure of technology. The project's PV was reduced from $909,100 to $818,000. You could have gotten the right answer by adding a fudge factor to the discount rate and discounting the original forecast of $1 million. But you have to think through the possible cash flows to get the fudge factor, and once you forecast the cash flows correctly, you don't need the fudge factor.

Fudge factors in discount rates are dangerous because they displace clear thinking about future cash flows. Here is an example.

EXAMPLE 9.2 ● Correcting for Optimistic Forecasts

The CFO of EZ² Corp. is disturbed to find that cash-flow forecasts for its investment projects are almost always optimistic. On average they are 10% too high. He therefore decides to compensate by adding 10% to EZ²'s WACC, increasing it from 12% to 22%.[17]

Suppose the CEO is right about the 10% upward bias in cash-flow forecasts. Can he just add 10% to the discount rate?

Project ZZ has level forecasted cash flows of $1,000 per year lasting for 15 years. The first two lines of Table 9.2 show these forecasts and their PVs discounted at 12%. Lines 3 and 4 show the corrected forecasts, each reduced by 10%, and the corrected PVs, which are (no surprise) also reduced by 10% (line 5). Line 6 shows the PVs when the uncorrected forecasts are discounted at 22%. The final line 7 shows the percentage reduction in PVs at the 22% discount rate, compared to the unadjusted PVs in line 2.

Line 5 shows the correct adjustment for optimism (10%). Line 7 shows what happens when a 10% fudge factor is added to the discount rate. The effect on the first year's cash flow is a PV "haircut" of about 8%, 2% less than the CFO expected. But later present values are knocked down by much more than 10%, because the fudge factor is compounded in the 22% discount rate. By years 10 and 15, the PV haircuts are 57% and 72%, far more than the 10% bias that the CFO started with.

Did the CFO really think that bias accumulated as shown in line 7 of Table 9.2? We doubt that he ever asked that question. If he was right in the first place, and the true bias is 10%, then adding a 10% fudge factor to the discount rate understates PV. The fudge factor also makes long-lived projects look much worse than quick-payback projects.[18]

Year:	1	2	3	4	5	...	10	...	15
1. Original cash-flow forecast	$1,000.00	$1,000.00	$1,000.00	$1,000.00	$1,000.00	...	$1,000.00	...	$1,000.00
2. PV at 12%	$892.90	$797.20	$711.80	$635.50	$567.40	...	$322.00	...	$182.70
3. Corrected cash-flow forecast	$900.00	$900.00	$900.00	$900.00	$900.00	...	$900.00	...	$900.00
4. PV at 12%	$803.60	$717.50	$640.60	$572.00	$510.70	...	$289.80	...	$164.40
5. PV correction	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	...	−10.0%	...	−10.0%
6. Original forecast discounted at 22%	$819.70	$671.90	$550.70	$451.40	$370.00	...	$136.90	...	$50.70
7. PV "correction" at 22% discount rate	−8.2%	−15.7%	−22.6%	−29.0%	−34.8%	...	−57.5%	...	−72.3%

▶ **TABLE 9.2**   The original cash-flow forecasts for the ZZ project (line 1) are too optimistic. The forecasts and PVs should be reduced by 10% (lines 3 and 4). But adding a 10% fudge factor to the discount rate reduces PVs by far more than 10% (line 6). The fudge factor overcorrects for bias and would penalize long-lived projects.

[17] The CFO is ignoring Brealey, Myers, and Allen's Second Law, which we cover in the next chapter.

[18] The optimistic bias could be worse for distant than near cash flows. If so, the CFO should make the time-pattern of bias explicit and adjust the cash-flow forecasts accordingly.

Discount Rates for International Projects

In this chapter we have concentrated on investments in the U.S. In Chapter 27 we say more about investments made internationally. Here we simply warn against adding fudge factors to discount rates for projects in developing economies. Such fudge factors are too often seen in practice.

It's true that markets are more volatile in developing economies, but much of that risk is diversifiable for investors in the U.S., Europe, and other developed countries. It's also true that more things can go wrong for projects in developing economies, particularly in countries that are unstable politically. Expropriations happen. Sometimes governments default on their obligations to international investors. Thus it's especially important to think through the downside risks and to give them weight in cash-flow forecasts.

Some international projects are at least partially protected from these downsides. For example, an opportunistic government would gain little or nothing by expropriating the local IBM affiliate, because the affiliate would have little value without the IBM brand name, products, and customer relationships. A privately owned toll road would be a more tempting target, because the toll road would be relatively easy for the local government to maintain and operate.

9-4 Certainty Equivalents—Another Way to Adjust for Risk

In practical capital budgeting, a single risk-adjusted rate is used to discount all future cash flows. This assumes that project risk does not change over time, but remains constant year-in and year-out. We know that this cannot be strictly true, for the risks that companies are exposed to are constantly shifting. We are venturing here onto somewhat difficult ground, but there is a way to think about risk that can suggest a route through. It involves converting the expected cash flows to **certainty equivalents.** First we work through an example showing what certainty equivalents are. Then, as a reward for your investment, we use certainty equivalents to uncover what you are really assuming when you discount a series of future cash flows at a single risk-adjusted discount rate. We also value a project where risk changes over time and ordinary discounting fails. Your investment will be rewarded still more when we cover options in Chapters 20 and 21 and forward and futures pricing in Chapter 26. Option-pricing formulas discount certainty equivalents. Forward and futures prices *are* certainty equivalents.

Valuation by Certainty Equivalents

Think back to the simple real estate investment that we used in Chapter 2 to introduce the concept of present value. You are considering construction of an office building that you plan to sell after one year for $420,000. That cash flow is uncertain with the same risk as the market, so $\beta = 1$. Given $r_f = 5\%$ and $r_m - r_f = 7\%$, you discount at a risk-adjusted discount rate of $5 + 1 \times 7 = 12\%$ rather than the 5% risk-free rate of interest. This gives a present value of $420,000/1.12 = \$375,000$.

Suppose a real estate company now approaches and offers to fix the price at which it will buy the building from you at the end of the year. This guarantee would remove any uncertainty about the payoff on your investment. So you would accept a lower figure than the uncertain payoff of $420,000. But how much less? If the building has a present value of $375,000 and the interest rate is 5%, then

$$PV = \frac{\text{Certain cash flow}}{1.05} = 375,000$$

$$\text{Certain cash flow} = \$393,750$$

In other words, a certain cash flow of $393,750 has exactly the same present value as an expected but uncertain cash flow of $420,000. The cash flow of $393,750 is therefore known as the *certainty-equivalent cash flow.* To compensate for both the delayed payoff and the uncertainty in real estate prices, you need a return of 420,000 − 375,000 = $45,000. One part of this difference compensates for the time value of money. The other part ($420,000 − 393,750 = $26,250) is a markdown or haircut to compensate for the risk attached to the forecasted cash flow of $420,000.

Our example illustrates two ways to value a risky cash flow:

Method 1: Discount the risky cash flow at a *risk-adjusted discount rate r* that is greater than r_f.[19] The risk-adjusted discount rate adjusts for both time and risk. This is illustrated by the clockwise route in Figure 9.3.

Method 2: Find the certainty-equivalent cash flow and discount at the risk-free interest rate r_f. When you use this method, you need to ask, What is the smallest *certain* payoff for which I would exchange the risky cash flow? This is called the *certainty equivalent,* denoted by CEQ. Since CEQ is the value equivalent of a safe cash flow, it is discounted at the risk-free rate. The certainty-equivalent method makes *separate* adjustments for risk and time. This is illustrated by the counterclockwise route in Figure 9.3.

We now have two identical expressions for the PV of a cash flow at period 1:[20]

$$PV = \frac{C_1}{1 + r} = \frac{CEQ_1}{1 + r_f}$$

For cash flows two, three, or *t* years away,

$$PV = \frac{C_t}{(1 + r)^t} = \frac{CEQ_t}{(1 + r_f)^t}$$

▶ FIGURE 9.3

Two ways to calculate present value. "Haircut for risk" is financial slang referring to the reduction of the cash flow from its forecasted value to its certainty equivalent.

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[19] The discount rate *r* can be less than r_f for assets with negative betas. But actual betas are almost always positive.

[20] CEQ_1 can be calculated directly from the capital asset pricing model. The certainty-equivalent form of the CAPM states that the certainty-equivalent value of the cash flow C_1 is $C_1 - \lambda$ cov $(\tilde{C}_1, \tilde{r}_m)$. Cov$(\tilde{C}_1, \tilde{r}_m)$ is the covariance between the uncertain cash flow, and the return on the market, $\tilde{r}_m$. Lambda, λ, is a measure of the market price of risk. It is defined as $(r_m - r_f)/\sigma_m^2$. For example, if $r_m - r_f = .08$ and the standard deviation of market returns is $\sigma_m = .20$, then lambda $= .08/.20^2 = 2$. We show on our Web site (**www.mhhe.com/bma**) how the CAPM formula can be restated in this certainty-equivalent form.

When to Use a Single Risk-Adjusted Discount Rate for Long-Lived Assets

We are now in a position to examine what is implied when a constant risk-adjusted discount rate is used to calculate a present value.

Consider two simple projects. Project A is expected to produce a cash flow of $100 million for each of three years. The risk-free interest rate is 6%, the market risk premium is 8%, and project A's beta is .75. You therefore calculate A's opportunity cost of capital as follows:

$$r = r_f + \beta (r_m - r_f)$$
$$= 6 + .75(8) = 12\%$$

Discounting at 12% gives the following present value for each cash flow:

Project A		
Year	**Cash Flow**	**PV at 12%**
1	100	89.3
2	100	79.7
3	100	71.2
	Total PV	240.2

Now compare these figures with the cash flows of project B. Notice that B's cash flows are lower than A's; but B's flows are safe, and therefore they are discounted at the risk-free interest rate. The *present value* of each year's cash flow is identical for the two projects.

Project B		
Year	**Cash Flow**	**PV at 6%**
1	94.6	89.3
2	89.6	79.7
3	84.8	71.2
	Total PV	240.2

In year 1 project A has a risky cash flow of 100. This has the same PV as the safe cash flow of 94.6 from project B. Therefore 94.6 is the certainty equivalent of 100. Since the two cash flows have the same PV, investors must be willing to give up $100 - 94.6 = 5.4$ in expected year-1 income in order to get rid of the uncertainty.

In year 2 project A has a risky cash flow of 100, and B has a safe cash flow of 89.6. Again both flows have the same PV. Thus, to eliminate the uncertainty in year 2, investors are prepared to give up $100 - 89.6 = 10.4$ of future income. To eliminate uncertainty in year 3, they are willing to give up $100 - 84.8 = 15.2$ of future income.

To value project A, you discounted each cash flow at the same risk-adjusted discount rate of 12%. Now you can see what is implied when you did that. By using a constant rate, you effectively made a larger deduction for risk from the later cash flows:

Year	Forecasted Cash Flow for Project A	Certainty-Equivalent Cash Flow	Deduction for Risk
1	100	94.6	5.4
2	100	89.6	10.4
3	100	84.8	15.2

The second cash flow is riskier than the first because it is exposed to two years of market risk. The third cash flow is riskier still because it is exposed to three years of market risk. This increased risk is reflected in the certainty equivalents that decline by a constant proportion each period.

Therefore, use of a constant risk-adjusted discount rate for a stream of cash flows assumes that risk accumulates at a constant rate as you look farther out into the future.

A Common Mistake

You sometimes hear people say that because distant cash flows are riskier, they should be discounted at a higher rate than earlier cash flows. That is quite wrong: We have just seen that using the same risk-adjusted discount rate for each year's cash flow implies a larger deduction for risk from the later cash flows. The reason is that the discount rate compensates for the risk borne *per period*. The more distant the cash flows, the greater the number of periods and the larger the *total* risk adjustment.

When You Cannot Use a Single Risk-Adjusted Discount Rate for Long-Lived Assets

Sometimes you will encounter problems where the use of a single risk-adjusted discount rate will get you into trouble. For example, later in the book we look at how options are valued. Because an option's risk is continually changing, the certainty-equivalent method needs to be used.

Here is a disguised, simplified, and somewhat exaggerated version of an actual project proposal that one of the authors was asked to analyze. The scientists at Vegetron have come up with an electric mop, and the firm is ready to go ahead with pilot production and test marketing. The preliminary phase will take one year and cost $125,000. Management feels that there is only a 50% chance that pilot production and market tests will be successful. If they are, then Vegetron will build a $1 million plant that would generate an expected annual cash flow in perpetuity of $250,000 a year after taxes. If they are not successful, the project will have to be dropped.

The expected cash flows (in thousands of dollars) are

$$C_0 = -125$$
$$C_1 = 50\% \text{ chance of } -1{,}000 \text{ and } 50\% \text{ chance of } 0$$
$$= .5(-1{,}000) + .5(0) = -500$$
$$C_t \text{ for } t = 2{,}3{,}\ldots = 50\% \text{ chance of } 250 \text{ and } 50\% \text{ chance of } 0$$
$$= .5(250) + .5(0) = 125$$

Management has little experience with consumer products and considers this a project of extremely high risk.[21] Therefore management discounts the cash flows at 25%, rather than at Vegetron's normal 10% standard:

$$\text{NPV} = -125 - \frac{500}{1.25} + \sum_{t=2}^{\infty} \frac{125}{(1.25)^t} = -125, \text{ or } -\$125{,}000$$

This seems to show that the project is not worthwhile.

Management's analysis is open to criticism if the first year's experiment resolves a high proportion of the risk. If the test phase is a failure, then there is no risk at all—the project is *certain* to be worthless. If it is a success, there could well be only normal risk from then on. That means there is a 50% chance that in one year Vegetron will have the opportunity to invest in a project of *normal* risk, for which the *normal* discount rate of 10% would be

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[21] We will assume that they mean high *market risk* and that the difference between 25% and 10% is *not* a fudge factor introduced to offset optimistic cash-flow forecasts.

Estimating Stock and Market Risk

▶ Spreadsheets such as Excel have some built-in statistical functions that are useful for calculating risk measures. You can find these functions by pressing *fx* on the Excel toolbar. If you then click on the function that you wish to use, Excel will ask you for the inputs that it needs. At the bottom left of the function box there is a Help facility with an example of how the function is used.

Here is a list of useful functions for estimating stock and market risk. You can enter the inputs for all these functions as numbers or as the addresses of cells that contain the numbers.

1. **VARP** and **STDEVP:** Calculate variance and standard deviation of a series of numbers, as shown in Section 7-2.
2. **VAR** and **STDEV:** Footnote 15 on page 164 noted that when variance is estimated from a sample of observations (the usual case), a correction should be made for the loss of a degree of freedom. VAR and STDEV provide the corrected measures. For any large sample VAR and VARP will be similar.
3. **SLOPE:** Useful for calculating the beta of a stock or portfolio.
4. **CORREL:** Useful for calculating the correlation between the returns on any two investments.
5. **RSQ:** R-squared is the square of the correlation coefficient and is useful for measuring the proportion of the variance of a stock's returns that can be explained by the market.
6. **AVERAGE:** Calculates the average of any series of numbers.

If, say, you need to know the standard error of your estimate of beta, you can obtain more detailed statistics by going to the *Tools* menu and clicking on *Data Analysis* and then on *Regression.*

SPREADSHEET QUESTIONS

The following questions provide opportunities to practice each of the Excel functions.

1. (VAR and STDEV) Choose two well-known stocks and download the latest 61 months of adjusted prices from **finance.yahoo.com**. Calculate the monthly returns for each stock. Now find the variance and standard deviation of the returns for each stock by using VAR and STDEV. Annualize the variance by multiplying by 12 and the standard deviation by multiplying by the square root of 12.
2. (AVERAGE, VAR, and STDEV) Now calculate the annualized variance and standard deviation for a portfolio that each month has equal holdings in the two stocks. Is the result more or less than the average of the standard deviations of the two stocks? Why?
3. (SLOPE) Download the Standard & Poor's index for the same period (its symbol is ^GSPC). Find the beta of each stock and of the portfolio. (*Note:* You need to enter the stock returns as the Y-values and market returns as the X-values.) Is the beta of the portfolio more or less than the average of the betas of the two stocks?
4. (CORREL) Calculate the correlation between the returns on the two stocks. Use this measure and your earlier estimates of each stock's variance to calculate the variance of a portfolio that is evenly divided between the two stocks. (You may need to reread Section 7-3 to refresh your memory of how to do this.) Check that you get the same answer as when you calculated the portfolio variance directly.
5. (RSQ) For each of the two stocks calculate the proportion of the variance explained by the market index. Do the results square with your intuition?
6. Use the *Regression* facility under the *Data Analysis* menu to calculate the beta of each stock and of the portfolio (beta here is called the coefficient of the X-variable). Look at the standard error of the estimate in the cell to the right. How confident can you be of your estimates of the betas of each stock? How about your estimate of the portfolio beta?

appropriate. Thus the firm has a 50% chance to invest $1 million in a project with a net present value of $1.5 million:

$$\text{Success} \rightarrow \text{NPV} = -1,000 + \frac{250}{.10} = +1,500 \; (50\% \text{ chance})$$

Pilot production
and market tests

$$\text{Failure} \rightarrow \text{NPV} = 0 \; (50\% \text{ chance})$$
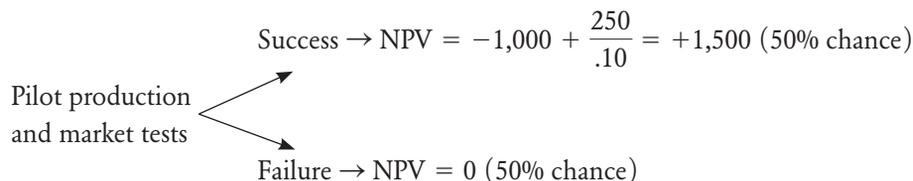
Thus we could view the project as offering an expected payoff of $.5(1,500) + .5(0) = 750$, or $750,000, at $t = 1$ on a $125,000 investment at $t = 0$. Of course, the certainty equivalent of the payoff is less than $750,000, but the difference would have to be very large to justify rejecting the project. For example, if the certainty equivalent is half the forecasted cash flow (an extremely large cash-flow haircut) and the risk-free rate is 7%, the project is worth $225,500:

$$\text{NPV} = C_0 + \frac{\text{CEQ}_1}{1 + r}$$

$$= -125 + \frac{.5(750)}{1.07} = 225.5, \text{ or } \$225,500$$

This is not bad for a $125,000 investment—and quite a change from the negative-NPV that management got by discounting all future cash flows at 25%.

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SUMMARY

In Chapter 8 we set out the basic principles for valuing risky assets. This chapter shows you how to apply those principles when valuing capital investment projects.

Suppose the project has the same market risk as the company's existing assets. In this case, the project cash flows can be discounted at the *company cost of capital*. The company cost of capital is the rate of return that investors require on a portfolio of all of the company's outstanding debt and equity. It is usually calculated as an after-tax *weighted-average cost of capital* (after-tax WACC), that is, as the weighted average of the after-tax cost of debt and the cost of equity. The weights are the relative market values of debt and equity. The cost of debt is calculated after tax because interest is a tax-deductible expense.

The hardest part of calculating the after-tax WACC is estimation of the cost of equity. Most large, public corporations use the capital asset pricing model (CAPM) to do this. They generally estimate the firm's equity beta from past rates of return for the firm's common stock and for the market, and they check their estimate against the average beta of similar firms.

The after-tax WACC is the correct discount rate for projects that have the same market risk as the company's existing business. Many firms, however, use the after-tax WACC as the discount rate for all projects. This is a dangerous procedure. If the procedure is followed strictly, the firm will accept too many high-risk projects and reject too many low-risk projects. It is *project* risk that counts: the true cost of capital depends on the use to which the capital is put.

Managers, therefore, need to understand why a particular project may have above- or below-average risk. You can often identify the characteristics of a high- or low-beta project even when the beta cannot be estimated directly. For example, you can figure out how much the project's cash flows are affected by the performance of the entire economy. Cyclical projects are generally high-beta projects. You can also look at operating leverage. Fixed production costs increase beta.

Don't be fooled by diversifiable risk. Diversifiable risks do not affect asset betas or the cost of capital, but the possibility of bad outcomes should be incorporated in the cash-flow forecasts. Also be careful not to offset worries about a project's future performance by adding a fudge factor to the discount rate. Fudge factors don't work, and they may seriously undervalue long-lived projects.

There is one more fence to jump. Most projects produce cash flows for several years. Firms generally use the same risk-adjusted rate to discount each of these cash flows. When they do this,

they are implicitly assuming that cumulative risk increases at a constant rate as you look further into the future. That assumption is usually reasonable. It is precisely true when the project's future beta will be constant, that is, when risk *per period* is constant.

But exceptions sometimes prove the rule. Be on the alert for projects where risk clearly does not increase steadily. In these cases, you should break the project into segments within which the same discount rate can be reasonably used. Or you should use the certainty-equivalent version of the DCF model, which allows separate risk adjustments to each period's cash flow.

The nearby box (on page 231) provides useful spreadsheet functions for estimating stock and market risk.

Michael Brennan provides a useful, but quite difficult, survey of the issues covered in this chapter:

M. J. Brennan, "Corporate Investment Policy," *Handbook of the Economics of Finance, Volume 1A, Corporate Finance,* eds. G. M. Constantinides, M. Harris, and R. M. Stulz (Amsterdam: Elsevier BV, 2003).

FURTHER READING

Select problems are available in McGraw-Hill Connect. Please see the preface for more information.

BASIC

PROBLEM SETS

1. Suppose a firm uses its company cost of capital to evaluate all projects. Will it underestimate or overestimate the value of high-risk projects?

2. A company is 40% financed by risk-free debt. The interest rate is 10%, the expected market risk premium is 8%, and the beta of the company's common stock is .5. What is the company cost of capital? What is the after-tax WACC, assuming that the company pays tax at a 35% rate?

3. Look back to the top-right panel of Figure 9.2. What proportion of Amazon's returns was explained by market movements? What proportion of risk was diversifiable? How does the diversifiable risk show up in the plot? What is the range of possible errors in the estimated beta?

4. Define the following terms:
    a. Cost of debt
    b. Cost of equity
    c. After-tax WACC
    d. Equity beta
    e. Asset beta
    f. Pure-play comparable
    g. Certainty equivalent

5. EZCUBE Corp. is 50% financed with long-term bonds and 50% with common equity. The debt securities have a beta of .15. The company's equity beta is 1.25. What is EZCUBE's asset beta?

6. Many investment projects are exposed to diversifiable risks. What does "diversifiable" mean in this context? How should diversifiable risks be accounted for in project valuation? Should they be ignored completely?

7. John Barleycorn estimates his firm's after-tax WACC at only 8%. Nevertheless he sets a 15% companywide discount rate to offset the optimistic biases of project sponsors and to

impose "discipline" on the capital-budgeting process. Suppose Mr. Barleycorn is correct about the project sponsors, who are in fact optimistic by 7% on average. Will the increase in the discount rate from 8% to 15% offset the bias?

8. Which of these projects is likely to have the higher asset beta, other things equal? Why?

   a. The sales force for project A is paid a fixed annual salary. Project B's sales force is paid by commissions only.

   b. Project C is a first-class-only airline. Project D is a well-established line of breakfast cereals.

9. True or false?

   a. The company cost of capital is the correct discount rate for all projects, because the high risks of some projects are offset by the low risk of other projects.

   b. Distant cash flows are riskier than near-term cash flows. Therefore long-term projects require higher risk-adjusted discount rates.

   c. Adding fudge factors to discount rates undervalues long-lived projects compared with quick-payoff projects.

10. A project has a forecasted cash flow of $110 in year 1 and $121 in year 2. The interest rate is 5%, the estimated risk premium on the market is 10%, and the project has a beta of .5. If you use a constant risk-adjusted discount rate, what is

    a. The PV of the project?

    b. The certainty-equivalent cash flow in year 1 and year 2?

    c. The ratio of the certainty-equivalent cash flows to the expected cash flows in years 1 and 2?

INTERMEDIATE

11. The total market value of the common stock of the Okefenokee Real Estate Company is $6 million, and the total value of its debt is $4 million. The treasurer estimates that the beta of the stock is currently 1.5 and that the expected risk premium on the market is 6%. The Treasury bill rate is 4%. Assume for simplicity that Okefenokee debt is risk-free and the company does not pay tax.

    a. What is the required return on Okefenokee stock?

    b. Estimate the company cost of capital.

    c. What is the discount rate for an expansion of the company's present business?

    d. Suppose the company wants to diversify into the manufacture of rose-colored spectacles. The beta of unleveraged optical manufacturers is 1.2. Estimate the required return on Okefenokee's new venture.

12. Nero Violins has the following capital structure:

Security	Beta	Total Market Value ($ millions)
Debt	0	$100
Preferred stock	.20	40
Common stock	1.20	299

   a. What is the firm's asset beta? (*Hint:* What is the beta of a portfolio of all the firm's securities?)

   b. Assume that the CAPM is correct. What discount rate should Nero set for investments that expand the scale of its operations without changing its asset beta? Assume a risk-free interest rate of 5% and a market risk premium of 6%.

13. The following table shows estimates of the risk of two well-known Canadian stocks:

	Standard Deviation, %	R^2	Beta	Standard Error of Beta
Toronto Dominion Bank	25	.25	.82	.18
Canadian Pacific	28	.30	1.04	.20

a. What proportion of each stock's risk was market risk, and what proportion was specific risk?

b. What is the variance of Toronto Dominion? What is the specific variance?

c. What is the confidence interval on Canadian Pacific's beta?

d. If the CAPM is correct, what is the expected return on Toronto Dominion? Assume a risk-free interest rate of 5% and an expected market return of 12%.

e. Suppose that next year the market provides a zero return. Knowing this, what return would you expect from Toronto Dominion?

14. You are given the following information for Golden Fleece Financial:

Long-term debt outstanding:	$300,000
Current yield to maturity (r_{debt}):	8%
Number of shares of common stock:	10,000
Price per share:	$50
Book value per share:	$25
Expected rate of return on stock (r_{equity}):	15%

Calculate Golden Fleece's company cost of capital. Ignore taxes.

15. Look again at Table 9.1. This time we will concentrate on Burlington Northern.

a. Calculate Burlington's cost of equity from the CAPM using its own beta estimate and the industry beta estimate. How different are your answers? Assume a risk-free rate of 5% and a market risk premium of 7%.

b. Can you be confident that Burlington's true beta is *not* the industry average?

c. Under what circumstances might you advise Burlington to calculate its cost of equity based on its own beta estimate?

16. What types of firms need to estimate industry asset betas? How would such a firm make the estimate? Describe the process step by step.

17. Binomial Tree Farm's financing includes $5 million of bank loans. Its common equity is shown in Binomial's *Annual Report* at $6.67 million. It has 500,000 shares of common stock outstanding, which trade on the Wichita Stock Exchange at $18 per share. What debt ratio should Binomial use to calculate its WACC or asset beta? Explain.

18. You run a perpetual encabulator machine, which generates revenues averaging $20 million per year. Raw material costs are 50% of revenues. These costs are variable—they are always proportional to revenues. There are no other operating costs. The cost of capital is 9%. Your firm's long-term borrowing rate is 6%.

Now you are approached by Studebaker Capital Corp., which proposes a fixed-price contract to supply raw materials at $10 million per year for 10 years.

a. What happens to the operating leverage and business risk of the encabulator machine if you agree to this fixed-price contract?

b. Calculate the present value of the encabulator machine with and without the fixed-price contract.

19. Mom and Pop Groceries has just dispatched a year's supply of groceries to the government of the Central Antarctic Republic. Payment of $250,000 will be made one year hence after the shipment arrives by snow train. Unfortunately there is a good chance of a coup d'état, in which case the new government will not pay. Mom and Pop's controller therefore decides to discount the payment at 40%, rather than at the company's 12% cost of capital.

    a. What's wrong with using a 40% rate to offset political risk?

    b. How much is the $250,000 payment really worth if the odds of a coup d'état are 25%?

20. An oil company is drilling a series of new wells on the perimeter of a producing oil field. About 20% of the new wells will be dry holes. Even if a new well strikes oil, there is still uncertainty about the amount of oil produced: 40% of new wells that strike oil produce only 1,000 barrels a day; 60% produce 5,000 barrels per day.

    a. Forecast the annual cash revenues from a new perimeter well. Use a future oil price of $50 per barrel.

    b. A geologist proposes to discount the cash flows of the new wells at 30% to offset the risk of dry holes. The oil company's normal cost of capital is 10%. Does this proposal make sense? Briefly explain why or why not.

21. A project has the following forecasted cash flows:

Cash Flows, $ Thousands			
C_0	C_1	C_2	C_3
−100	+40	+60	+50

    The estimated project beta is 1.5. The market return r_m is 16%, and the risk-free rate r_f is 7%.

    a. Estimate the opportunity cost of capital and the project's PV (using the same rate to discount each cash flow).

    b. What are the certainty-equivalent cash flows in each year?

    c. What is the ratio of the certainty-equivalent cash flow to the expected cash flow in each year?

    d. Explain why this ratio declines.

22. The McGregor Whisky Company is proposing to market diet scotch. The product will first be test-marketed for two years in southern California at an initial cost of $500,000. This test launch is not expected to produce any profits but should reveal consumer preferences. There is a 60% chance that demand will be satisfactory. In this case McGregor will spend $5 million to launch the scotch nationwide and will receive an expected annual profit of $700,000 in perpetuity. If demand is not satisfactory, diet scotch will be withdrawn.

    Once consumer preferences are known, the product will be subject to an average degree of risk, and, therefore, McGregor requires a return of 12% on its investment. However, the initial test-market phase is viewed as much riskier, and McGregor demands a return of 20% on this initial expenditure.

    What is the NPV of the diet scotch project?

CHALLENGE

23. Suppose you are valuing a future stream of high-risk (high-beta) cash *outflows*. High risk means a high discount rate. But the higher the discount rate, the less the present value. This seems to say that the higher the risk of cash outflows, the less you should worry about them! Can that be right? Should the sign of the cash flow affect the appropriate discount rate? Explain.

24. An oil company executive is considering investing $10 million in one or both of two wells: well 1 is expected to produce oil worth $3 million a year for 10 years; well 2 is expected to produce $2 million for 15 years. These are *real* (inflation-adjusted) cash flows.

The beta for *producing wells* is .9. The market risk premium is 8%, the nominal risk-free interest rate is 6%, and expected inflation is 4%.

The two wells are intended to develop a previously discovered oil field. Unfortunately there is still a 20% chance of a dry hole in each case. A dry hole means zero cash flows and a complete loss of the $10 million investment.

Ignore taxes and make further assumptions as necessary.

a. What is the correct real discount rate for cash flows from developed wells?

b. The oil company executive proposes to add 20 percentage points to the real discount rate to offset the risk of a dry hole. Calculate the NPV of each well with this adjusted discount rate.

c. What do *you* say the NPVs of the two wells are?

d. Is there any *single* fudge factor that could be added to the discount rate for developed wells that would yield the correct NPV for both wells? Explain.

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REAL-TIME DATA ANALYSIS

You can download data for the following questions from Standard & Poor's Market Insight Web site (**www.mhhe.com/edumarketinsight**)—see the "Monthly Adjusted Prices" spreadsheet—or from **finance.yahoo.com**.

1. Look at the companies listed in Table 8.2. Calculate monthly rates of return for two successive five-year periods. Calculate betas for each subperiod using the Excel SLOPE function. How stable was each company's beta? Suppose that you had used these betas to estimate expected rates of return from the CAPM. Would your estimates have changed significantly from period to period?

2. Identify a sample of food companies. For example, you could try Campbell Soup (CPB), General Mills (GIS), Kellogg (K), Kraft Foods (KFT), and Sara Lee (SLE).

    a. Estimate beta and R^2 for each company, using five years of monthly returns and Excel functions SLOPE and RSQ.

    b. Average the returns for each month to give the return on an equally weighted portfolio of the stocks. Then calculate the industry beta using these portfolio returns. How does the R^2 of this portfolio compare with the average R^2 of the individual stocks?

    c. Use the CAPM to calculate an average cost of equity (r_{equity}) for the food industry. Use current interest rates—take a look at the end of Section 9-2—and a reasonable estimate of the market risk premium.

STANDARD &POOR'S

MINI-CASE ● ● ● ● ●

The Jones Family, Incorporated

The Scene: Early evening in an ordinary family room in Manhattan. Modern furniture, with old copies of *The Wall Street Journal* and the *Financial Times* scattered around. Autographed photos of Alan Greenspan and George Soros are prominently displayed. A picture window reveals a distant view of lights on the Hudson River. John Jones sits at a computer terminal, glumly sipping a glass of chardonnay and putting on a carry trade in Japanese yen over the Internet. His wife Marsha enters.

Marsha: Hi, honey. Glad to be home. Lousy day on the trading floor, though. Dullsville. No volume. But I did manage to hedge next year's production from our copper mine. I couldn't get a good quote on the right package of futures contracts, so I arranged a commodity swap.

John doesn't reply.

Marsha: John, what's wrong? Have you been selling yen again? That's been a losing trade for weeks.

John: Well, yes. I shouldn't have gone to Goldman Sachs's foreign exchange brunch. But I've got to get out of the house somehow. I'm cooped up here all day calculating covariances and efficient risk-return trade-offs while you're out trading commodity futures. You get all the glamour and excitement.

Marsha: Don't worry, dear, it will be over soon. We only recalculate our most efficient common stock portfolio once a quarter. Then you can go back to leveraged leases.

John: You trade, and I do all the worrying. Now there's a rumor that our leasing company is going to get a hostile takeover bid. I knew the debt ratio was too low, and you forgot to put on the poison pill. And now you've made a negative-NPV investment!

Marsha: What investment?

John: That wildcat oil well. Another well in that old Sourdough field. It's going to cost $5 million! Is there any oil down there?

Marsha: That Sourdough field has been good to us, John. Where do you think we got the capital for your yen trades? I bet we'll find oil. Our geologists say there's only a 30% chance of a dry hole.

John: Even if we hit oil, I bet we'll only get 150 barrels of crude oil per day.

Marsha: That's 150 barrels day in, day out. There are 365 days in a year, dear.

John and Marsha's teenage son Johnny bursts into the room.

Johnny: Hi, Dad! Hi, Mom! Guess what? I've made the junior varsity derivatives team! That means I can go on the field trip to the Chicago Board Options Exchange. (*Pauses.*) What's wrong?

John: Your mother has made another negative-NPV investment. A wildcat oil well, way up on the North Slope of Alaska.

Johnny: That's OK, Dad. Mom told me about it. I was going to do an NPV calculation yesterday, but I had to finish calculating the junk-bond default probabilities for my corporate finance homework. (*Grabs a financial calculator from his backpack.*) Let's see: 150 barrels a day times 365 days per year times $50 per barrel when delivered in Los Angeles . . . that's $2.7 million per year.

John: That's $2.7 million *next* year, assuming that we find any oil at all. The production will start declining by 5% every year. And we still have to pay $10 per barrel in pipeline and tanker charges to ship the oil from the North Slope to Los Angeles. We've got some serious operating leverage here.

Marsha: On the other hand, our energy consultants project increasing oil prices. If they increase with inflation, price per barrel should increase by roughly 2.5% per year. The wells ought to be able to keep pumping for at least 15 years.

Johnny: I'll calculate NPV after I finish with the default probabilities. The interest rate is 6%. Is it OK if I work with the beta of .8 and our usual figure of 7% for the market risk premium?

Marsha: I guess so, Johnny. But I am concerned about the fixed shipping costs.

John: (*Takes a deep breath and stands up.*) Anyway, how about a nice family dinner? I've reserved our usual table at the Four Seasons.

Everyone exits.

Announcer: Is the wildcat well really negative-NPV? Will John and Marsha have to fight a hostile takeover? Will Johnny's derivatives team use Black–Scholes or the binomial method? Find out in the next episode of The Jones Family, Incorporated.

You may not aspire to the Jones family's way of life, but you will learn about all their activities, from futures contracts to binomial option pricing, later in this book. Meanwhile, you may wish to replicate Johnny's NPV analysis.

QUESTIONS

1. Calculate the NPV of the wildcat oil well, taking account of the probability of a dry hole, the shipping costs, the decline in production, and the forecasted increase in oil prices. How long does production have to continue for the well to be a positive-NPV investment? Ignore taxes and other possible complications.

2. Now consider operating leverage. How should the shipping costs be valued, assuming that output is known and the costs are fixed? How would your answer change if the shipping costs were proportional to output? Assume that unexpected fluctuations in output are zero-beta and diversifiable. (*Hint:* The Jones's oil company has an excellent credit rating. Its long-term borrowing rate is only 7%.)